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DESIGN AND DECOMPOSITION OF WASTE PROGNOSTIC MODEL WITH HIERARCHICAL STRUCTURES

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Abstract: *The waste management is a dynamically progressive area, with the current trend leading to circular economy scheme. The development in this area requires quality prognosis reflecting the analysed timeframe. The forecast of the waste production and composition of waste is an important aspect with regards to the planning in waste management. However, the regular prognostic methods are not appropriate for these purposes due to short time series of historical data and unavailability of socio-economic data. The paper proposes a general approach via mathematical model for forecasting of future waste-related parameters based on spatially distributed data with hierarchical structure. The approach is based on principles of regression analysis with final balance to ensure the compliance of aggregated data values. The selection of the regression function is a part of mathematical model for high-quality description of data trend. In addition, outlier values are cleared, which occur abundantly in the database. The decomposition of the model into subtasks is performed in order to simpler implementation and reasonable time solvability. The individual algorithm steps are applied to municipal waste production data in the Czech Republic.*

Keywords: *waste production, forecasting, prognostic model, short time series, regression analysis, nonlinear regression*

1 Introduction

The waste management is a dynamically progressive area. In developed countries, the trend is to move from traditional waste treatment to the circular economy. The aim is to maximize the use of waste as a secondary material thus exploit the potential hidden in a waste. The study [1] addresses this issue in more detail and also suggests the practical application of the circular economy. For a possible transition to the circulatory economy, the processing infrastructure needs to be adequately adapted. To such changes must especially processing plants timely respond. Construction of some kinds of new facilities is a time-consuming process [2]. Therefore, it is necessary to built quality prognosis reflecting the required timeframe, among the most important are the production and composition of waste.

The paper [3] deals with a summary of previously published models for estimating current and future production of mixed municipal waste (MMW). There are mentioned 45 modeling approaches and a system for selecting the appropriate analysis method is designed. In most cases the correlation and regression analysis seem to be a suitable approach, the application of time series analysis is recommended in special cases e.g for providing information about seasonal character. The study [4] also provides a review of models for MMW production, methods based on artificial intelligence have a significant presence here. For example, the paper [5] utilizes the fuzzy inference system for the calculation of the urban solid waste. Nevertheless, most models use a regression analysis to describe the relationship between socio-economic factors and waste generation. Unfortunately, assumptions of regression analysis are often not met for real data. In some cases, these conditions may be replaced by more general terms and a generalized regression model is built [6]. Most models are based on socio-economic data, but such a database is not always available at microregional level. These models are not generally suitable for some applications, and their use for the data processed in this work is very limited.

Very often, the only explained parameter of the waste generation data is the unit of time, in such a case the time series analysis is the only option. The paper [7] compares time series models for forecasting the amount of municipal waste, but these models are only applicable for sufficiently long time series. In short time series, it is not possible to find autocorrelation in the data or to identify individual time series components (seasonal, cyclic, random). Nevertheless, the trend component often has a characteristic course even in short time series.

The work [8] deals with forecasting of MMW production with respect to the case study for the Romanian city of Iasi. The authors use the Waste Prognostic Tool [9] and Minitab in combination with regression analysis and time series analysis. As the most suitable trend model appears to be S-curves both for the total production of MMW and for individual components of waste in MMW.

This work proposes a methodology for forecasting waste production on the basis of the annual data available for a very short time series. This is done for different levels of detail (country level, regional level, microregional level). Traditional prognostic procedures fail because of unavailability socio-economic data at microregional level. Thus, the problem of waste amount identification is introduced through a mixed integer non-linear programming model, which maintains the relations arising from the character of the task. However, the regular methods for solving this model fail due to its complexity. The heuristic approaches come in such cases, while the decomposition of the problem into smaller parts is commonly used. The procedure uses the principles of regression analysis to model the trend in historical data, where the only independent variable is the time. Consistency between predicted values of waste generation is ensured by the final balance in the reference year. The hierarchy of territorial units and different waste code numbers are maintained for such adjusted values. The presented approach can be applied to arbitrary data with a spatial or type hierarchical structure.

2 Mathematical Model

In this section, prognostic model based on principles of regression analysis will be presented. The model determines the optimal choice for trend function with outliers detection and final balance at the same time. This model consists of two parts, where the results of the first stage are used for the second stage. Details of these two parts are described in the following subsections. First of all, the necessary sets, parameters and decision variables are defined.

sets and indices

$i \in I$	index determining year of the forecast	$j, z \in J$	index of data set, $J \subset I$
$s \in S$	index determining aggregation of territorial unit	$n \in N$	index of territorial unit
$t \in T$	index determining aggregation of catalog numbers	$h \in H$	index of waste catalog number
$k \in K$	index of regression function		

parameters

$d_{j,n,h}$	production of waste h in year j in the territory n	$y_{j,s,t}$	aggregation of data $d_{j,n,h}$
$\delta_{n,s}$	matrix reflecting the hierarchy of territorial units	$v_{s,t}$	weight of aggregation
$\gamma_{h,t}$	matrix reflecting the hierarchy of catalog numbers	M	value much higher than all conditions
$w_{s,t,z}$	weight of aggregation with omitted data z	$l_{j,z}$	0-1 indicator of omission data z
p_k	number of regression parameters in function k	$Q_{s,t}$	critical value of the criterion $D_{s,t,z}$
$B_{s,t,k}$	indicator of chosen regression function	$E_{j,s,t}$	indicator of outliers

$f_{i,s,t,k}(\mathbf{x})$	regression function k for groups s and t with independent variable i and parameters $\mathbf{x}$
$f^*_{i,s,t,k,z}(\mathbf{x})$	regression function k for omitted data z, groups s and t with independent variable i and regression parameters $\mathbf{x}$

decision variables

$m_{i,s,t}$	estimate of production based on aggregation s and t in the year i
$\mathbf{x}$	vector of regression parameters, $\dim(\mathbf{x}) = \max(p_k)$
$b_{s,t,k}$	binary variable, 1 for chosen function, otherwise 0
$D_{s,t,z}$	value of criterion with omitted point z
$Se^2_{s,t}$	estimate of residual variance
$m^*_{j,s,t,z}$	estimate of production based on aggregation s and t in the year i and omitted data z
$\varepsilon_{s,t,z}$	binary variable, 0 if $D_{s,t,z} < Q_{s,t}$, otherwise 1
$\alpha_{s,t}$	binary variable, 1 for decreasing trend, 0 for increasing trend

2.1 Regression Function Selection and Detection of Outliers

The first part of the model consists of regression function selection and detection of outliers. The quality model can only be created with the appropriate regression function $f_{i,s,t,k}(\mathbf{x})$, which has the ability to describe trend in historical data. On the assumption that the individual time series may be characterized by a completely different course, the possibility of selecting the regression function is preserved. The binary variable $b_{s,t,k}$ always activates the only one function from the set K. This part of mathematical model also includes identification of outliers through Cook's distance [10].

Objective function

The objective function (1) minimizes the sum of squared errors, where $v_{s,t}$ and $w_{s,t,z}$ normalize balancing data. The parameter $y_{j,s,t}$ determines aggregation of data, $m_{j,s,t}$ and $m^*_{j,s,t,z}$ are estimations of production. The objective function consists of two parts. First of them makes estimation based on the complete dataset and it is multiplied by a large value M. This measure puts a higher priority on this section and therefore the regression function is selected for complete time series. The second part works with omitted data by vector $l_{j,z}$, e.g. for omitting the first point $l_{j,1} = \underbrace{(0,1,1,1,...,1)}_{|J|}$.

$$\min \quad M\sum_{s\in S}\sum_{t\in T}\sum_{j\in J} v_{s,t}(m_{j,s,t} - y_{j,s,t})^2 + \sum_{s\in S}\sum_{t\in T}\sum_{j\in J}\sum_{z\in J} l_{j,z}w_{s,t,z}(m^*_{j,s,t,z} - y_{j,s,t})^2. \tag{1}$$

Constraints

The aggregated $y_{j,s,t}$ data are formed in the condition (2) to create territorial s and waste groups t from original data $d_{j,n,h}$. The model admits all possible aggregations, so the matrix $\delta_{n,s}$ defines all pairs, triples etc. of territories. The aggregations of waste types are constructed in the same way through the matrix $\gamma_{h,t}$.

$$y_{j,s,t} = \sum_{h\in H}(\sum_{n\in N} d_{j,n,h}\delta_{n,s})\gamma_{h,t}, \forall j \in J, \forall s \in S, \forall t \in T. \tag{2}$$

The waste production can have completely different character for individual units. The model retains the choice of the regression function $f_{i,s,t,k}(\mathbf{x})$ from the set K (3), using binary variable $b_{s,t,k}$ (5). Because the sum of $b_{s,t,k}$ over k is equal to one (4), the only one regression function is active for calculation of estimation $m_{i,s,t}$.

$$m_{i,s,t} = \sum_{k\in K} f_{i,s,t,k}(\mathbf{x})b_{s,t,k}, \forall i \in I, \forall s \in S, \forall t \in T, \tag{3}$$

$$\sum_{k\in K} b_{s,t,k} = 1, \forall s \in S, \forall t \in T, \tag{4}$$

$$b_{s,t,k} \in \{0,1\}, \forall s \in S, \forall k \in K, \forall t \in T. \tag{5}$$

The minimum $u_{s,t}$ and maximum $U_{s,t}$ production limits prevent unrealistic changes in production in the period under review I, (6).

$$u_{s,t} \leq m_{i,s,t} \leq U_{s,t}, \forall i \in I, \forall s \in S, \forall t \in T. \tag{6}$$

The functions contained in the set K do not have generally a monotonic course, but due to the dataset, this property is required. The condition (7) ensures a monotonous trend, where binary variable $\alpha_{s,t}$ (8) takes the value 0 for increasing trend and 1 for decreasing trend.

$$(2\alpha_{s,t} - 1)(m_{i+1,s,t} - m_{i,s,t}) \leq 0, \forall s \in S, \forall t \in T, \forall i = 1,..,|I| - 1, \tag{7}$$

$$\alpha_{s,t} \in \{0,1\}, \forall s \in S, \forall t \in T. \tag{8}$$

Appropriate measure for detecting influential observation in the case of linear and nonlinear regression is Cook's distance (9) [10]. This criterion is based on tracking regression model changes after skipping individual points from dataset. The squared difference between models is normalized by estimating the variance of residues and the number of parameters of regression function.

$$D_{s,t,z} = \frac{\sum_{j\in J}(m_{j,s,t} - m^*_{j,s,t,z})^2}{Se^2_{s,t}\sum_{k\in K} p_k b_{s,t,k}}, \forall s \in S, \forall t \in T, \forall z \in J. \tag{9}$$

The constraints (10)-(14) ensure computation of values, which are needed for Cook's distance calculation. The regression models $m^*_{j,s,t,z}$ with omitted point z are created using already selected function (10). Equation (11) limits minimum and maximum waste production, (12) and (13) again maintains a monotonous trend. The estimation of the residues variance is calculated in (14).

$$m^*_{j,s,t,z} = \sum_{k\in K} f^*_{j,s,t,k,z}(\mathbf{x})b_{s,t,k}, \forall j \in J, \forall s \in S, t \in T, \forall z \in J, \tag{10}$$

$$u_{s,t} \leq m^*_{i,s,t,z} \leq U_{s,t}, \forall i \in I, \forall s \in S, \forall t \in T, \tag{11}$$

$$(2\alpha_{s,t,z} - 1)(m^*_{i+1,s,t,z} - m^*_{i,s,t,z}) \leq 0, \forall s \in S, \forall t \in T, \forall z \in J, \forall i = 1, .., |I| - 1, \tag{12}$$

$$\alpha_{s,t,z} \in \{0,1\}, \forall s \in S, \forall t \in T, \forall z \in J, \tag{13}$$

$$Se^2_{s,t} = \frac{1}{|J|}\sum_{j\in J}\left((m_{j,s,t} - y_{j,s,t}) - \frac{\sum_{j\in J}(m_{j,s,t} - y_{j,s,t})}{|J|}\right)^2, \forall s \in S, \forall t \in T. \tag{14}$$

The Cook's distance $D_{s,t,z}$ is compared with the critical value $Q_{s,t}$, which is in most cases equal to 1 [11]. In the part (15)-(17) the binary variable $\varepsilon_{s,t,z}$ is equal to 1, if the critical value was exceeded. The outliers are marked in this way.

$$D_{s,t,z} \leq Q_{s,t} + M\varepsilon_{s,t,z}, \forall s \in S, \forall t \in T, \forall z \in J, \tag{15}$$

$$D_{s,t,z} > Q_{s,t}\varepsilon_{s,t,z}, \forall s \in S, \forall t \in T, \forall z \in J, \tag{16}$$

$$\varepsilon_{s,t,z} \in \{0,1\}, \forall s \in S, \forall z \in J, \forall t \in T. \tag{17}$$

For the second part of the model, variables $b_{s,t,k}$ and $\varepsilon_{s,t,z}$ become the parameters according to (18) and (19). The informations about selected regression function and outliers are transferred to the second section of the model.

$$B_{s,t,k} = b_{s,t,k}, \forall s \in S, \forall t \in T, \forall k \in K, \tag{18}$$

$$E_{j,s,t} = (1 - \varepsilon_{s,t,z}), \forall s \in S, \forall t \in T, \forall j \in J, \forall z \in J. \tag{19}$$

2.2 Trend Model in Data and Final Balance

The second part of model determines the trend in historical data using already selected regression function. The outliers are deleted in this section without replacement, see 2.1. In addition, a balance of the simple trend is formed with respect to the hierarchical structure. So amount in higher unit is equal to the sum of amounts in all units, which the higher one consists of. This ensures consistency in aggregate data.

Objective function

The weighted sum of squared errors is again minimized, the aggregated data $y_{j,s,t}$ are given from the previous section. The parameter $E_{j,s,t}$ is equal to 0 for all outliers, so these detected values have no effect on the objective function.

$$\min \sum_{s\in S}\sum_{t\in T}\sum_{j\in J} E_{j,s,t}v_{s,t}(m_{j,s,t} - y_{j,s,t})^2. \tag{20}$$

Constraints

By (21), the regression model is created using regression functions, which were selected in the previous part of the mathematical model 2.1. So $B_{s,t,k}$ is not a binary variable but a fixed parameter. The production estimation is also in this case within the limits $u_{s,t}$ and $U_{s,t}$, (22).

$$m_{i,s,t} = \sum_{k\in K} f_{i,s,t,k}(\mathbf{x})B_{s,t,k}, \forall i \in I, \forall s \in S, t \in T, \tag{21}$$

$$u_{s,t} \leq m_{i,s,t} \leq U_{s,t}, \forall i \in I, \forall s \in S, \forall t \in T. \tag{22}$$

Each lower territorial element is part of another territorial area. An estimate of production in a given territory must correspond to the sum of the production in all the territories belonging to it. The same applies to aggregated data of waste catalog number. This consistency in the hierarchical structure is ensured for the year $i_0 \in I$ by correcting of estimates in terms (23) and (24).

$$m_{i_0,s,t} = \sum_{n \in N} m_{i_0,n,t}\delta_{n,s}, \forall s \in S, \forall t \in T, \tag{23}$$

$$m_{i_0,s,t} = \sum_{h \in H} m_{i_0,s,h}\gamma_{h,t}, \forall s \in S, \forall t \in T. \tag{24}$$

The design of the mathematical model, as described in this section, is the problem of mixed integer nonlinear programming. A system of decomposition has been proposed to ensure the solvability of this problem.

3 Decomposition of Global Problem

Since the previous mathematical model represents a complicated problem, which contains hundreds of thousands of continuous and especially binary variables, it is split into sub-tasks to ensure the real-time solvability. The data aggregation, trend analysis and detection of outliers are solved gradually instead of one extensive problem and the solution serves as the input to Justine [12]. Complete procedure is illustrated in the Fig. 1. Data could be an input for "Global problem" or alternatively go through the parts of aggregation, trend analysis, outliers and finally Justine. If an outlying point is found, it is deleted and the new trend model is calculated with the regression function which was selected for data without skipping of outliers.

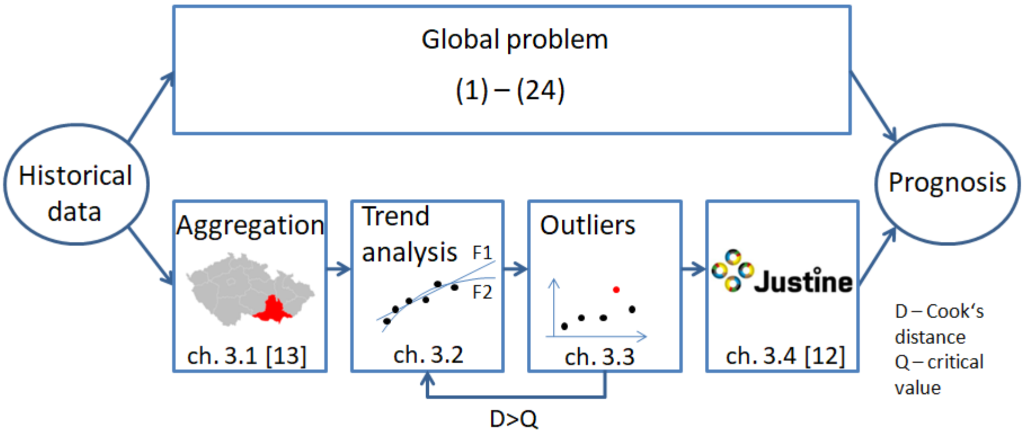

Figure 1: Illustration of the decomposition procedure

The individual steps are applied to the municipal waste production data in the Czech Republic at three levels of territorial division: microregions, regions, the Czech Republic. The application discusses the following groups of waste:

- Selected components of municipal waste
- Certain sorted fractions of waste (paper, plastic, glass)
- Composition of MMW
- Total production of certain sorted fractions of waste (paper, plastic, glass)

3.1 Aggregation

The subject of aggregation was in the interest of the authors of the paper [13]. The aggregated groups were created to produce the best possible data in the sense of smallest residual squares. This modification ensures a reduction of the number of groups compared to global model in constraint (2).

3.2 Trend Analysis

The model of trend in historical data is created with the use of regression analysis where the time is the only one independent variable. Attention is paid to selection of regression functions and determination of initial values, because we are dealing with nonlinear regression.

Functions selection

The mathematical model in the section 2 preserves the possibility of regression function selection for each time series through constraint (3). The goal is to create a list of regression functions that characterize tha data trend well thus significantly reduce the amount of functions from the set K. Analysis of waste production is tested for 12 types of waste in the Czech Republic on a microregional level (206 microregions), which correspond to 2,472 time series that require a regression model. This time-consuming task could be more effective using the cluster analysis and work with representatives only. The methodology was described at [14] using TableCurve 2D software [15], for the municipal waste the function list consists of 10 various regression functions, see Tab. 1.

Table 1: Selected regression functions

1. $y = a + bx^c$	2. $\ln(y) = a + b\exp(-x)$	3. $y = a + \frac{b}{1+e(\frac{x-c}{d})^2\exp\left((1-e)0.5(\frac{x-c}{d})^2\right)}$	4. $y = a + b\frac{(\arctan(\frac{x-c}{d})+\frac{\pi}{2})}{\pi}$
5. $y = a + b\exp(-x)$	6. $y^{0.5} = a + b\exp(-x)$	7. $y = a + \frac{b}{\left(1+\frac{(x-c)^2}{d^2 e}\right)^{\frac{d}{2}+\frac{1}{2}}}$	
8. $y^{-1} = a + b\exp(-x)$	9. $y = a + \frac{b}{1+\exp\left(\frac{-(x-c)}{d}\right)}$	10. $y = a + b\exp\left(-\exp\left(\frac{-(x-d\ln(\ln(2))-c)}{d}\right)\right)$	

The historical data is extrapolated by function, which achieves the highest determination coefficient. Simultaneously, the condition of unchanging monotony on the set I must be satisfiend under (7).

Percentage use of regression functions for trend in historical data from all time series summarizes Tab. 2. Function number 9 significantly exceeds all others, this function belongs to S-curves.

Table 2: Percentage utilization of regression functions

Function	1	2	3	4	5	6	7	8	9	10
Amount of time series [%]	7.39	0.28	2.07	0.34	0.15	0.11	14.44	0.11	75.20	0.11

Initial values

The selected regression functions clearly indicate nonlinear regression, unfortunately the minimum error can not be specified explicitly. Therefore, the initial values of the regression parameters are required for numerical procedures. The quality initial estimates lead to finding solution faster and more reliably.

The parameter a, in each of the above regression functions, allows the shift the model along the vertical axis. For this reason, the initial estimate of parameter a is set to the value of last point in the time series. In the case that parameters do not represent some empirical dependence and the function are not linearizable, the paper [16] suggests searching in the grid. The extensive coverage of possible parameters and their combination is generated. Each of these combinations is evaluated and the initial values are determined by the variant with the lowest value of the objective function. Valuable information about approximate regression parameters is output from the software TableCurve 2D. Simultaneously with its use to select the regression function, regression parameters are recorded for representatives of clusters. The minimal and maximal values of each parameter for the representatives are set as the boundaries for its initial value. Nevertheless, the grid is still to extensive so instead of testing all parameters combinations, the initial values are randomly generated from uniform distribution from the interval of minimal and maximal parameter for representatives.

3.3 Outlier Detection

The presence of extreme values and outliers can cause distortion of results so their identification is an important assumption of model quality. Waste production data are recorded every year and interval in the years 2009 - 2015 is available for this study. The extreme values do not occur in database and we will focus on outlier detection i.e fluctuations in waste production. The Cook's distance is suitable criterion for utilization in linear and nonlinear regression, this issue is included in mathematical model in part (9)-(17). This measure is based on tracking model changes when omitting individual points.

If the Cook's distance exceeds the critical value, the respective point is marked as influential and is most likely an outlier. Traditionally, the critical value for nonlinear regression is equal to 1. But according to [17], the better interpretation is, that point is influential, if the Cook's distance at this point is significantly higher than at others in the same time series. This high Cook's distances are revealed using Q-test,

$$Q_n = \frac{x_n - x_{n-1}}{x_n - x_1}, \tag{25}$$

where x_i are sorted elements of the tested set $x_1 < x_2 < \ldots < x_n$. If Q_n exceeds tabulated critical value $\overline{Q}_{n,\alpha}$ on the level of significance α, this point of the respective time series is outlier. Attention should be paid to the marginal points of the time series, which often have a high impact on the shape of the model. The Cook's distance for this point overcomes others, as Fig. 2 shows, even though it is not outlier. A total of 25.41 % starting points of time series are marked as influential points and 15.76 % of end points. Omitting these points is not in the interest of prognosis, because it could lose information about the trend. Therefore, the distance at these points is supplemented by a residue analysis. The value is suspicious of remoteness, if the residue is significantly greater than the others in the same time serie. Assuming a normal distribution of residues $\widehat{\varepsilon}_i$, $\widehat{\varepsilon}_i \sim N(0, \sigma^2)$, 95 % of these residues lie in the interval $(-2\sigma, 2\sigma)$. If the residue at the marginal point is outside this interval, and simultaneously the point is marked by Cook's distance, it is deleted as an outlier. For internal time series points, outlier identification is left only for Cook's distance that works well here.

Based on the Cook's distance, which is covered by residue analysis in the marginal values, the proposed methodology deletes 4.80 % points from all available data. There in no need to delete any of the points in 66.93 % of time series, for the remaining time series, one or two outliers are identified.

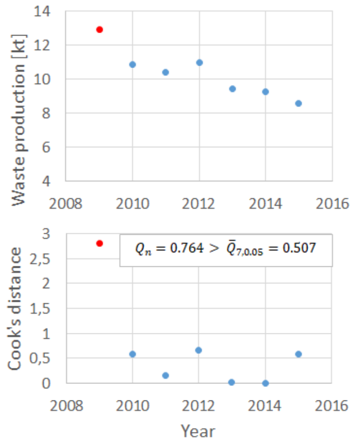

Figure 2: Cook's distance for waste production data

3.4 Resulting Model and Final Balance

In case a deletion of outlier, the trend model is determined again without the values being removed, as a constraint (21). The last step of the methodology described in the Fig. 1 is the balance using tool Justine [12]. Thus the consistency of the predicted value in the chosen year with respect to the hierarchical structure of the data is guaranteed. The constraints (23) and (24) include this balance in the mathematical model.

4 Results

The computation in the form of decomposition (section 3) is implemented in the system GAMS (The General Algebraic Modeling System). The forecast of municipal waste, whose outputs are summed up in this chapter, arises for three re-generated scenarios of initial estimates. The Tab. 3 summarizes the task in terms of time requirement and amount of variables. [1]

Table 3: Characteristics of the approach in terms of implementation

	Number of tasks nonlin. prog.	Total amount of regr. parameters	Duration (h)
Function selection	79,560	262,548	19.81
Outliers	55,692	32,538	13.86
Resulting model	3,054	12,645	0.76

Solving a mathematical model in the form of decomposition certainly leads to an error in the calculations we make in this approach. For the part dedicated to aggregation, this deviation is not significant, because the set of all possible aggregation is limited to cases that produce the highest quality data. A similar situation occurs in the analysis of the trend, the set of regression functions is narrowed to the best data model. The using of cluster analysis can cause errors because the regression functions are selected for cluster representatives. Identification of outliers for global problem and decomposed form has the same principle. Another deviation from global model is in the part of Justine balance. In the case of complete model, this balance is in the trend modeling phase and the model is directly affected by this balance. A quantification of errors caused by the application of a decomposed task is the subject of further development.

[1]Computations were realized on the computer with Microsoft Windows 10 Home 64-bit, Intel(R)Core(TM)i7-5500U CPU @2.40GHz 2.40 GHz, 8 GB of RAM.

5 Conclusion

This paper deals with the development of prognostic models based on historical data with hierarchical structure. The proposal of the mathematical model uses principles of regression analysis and the opportunity to select regression function for each time series is retained. In the first part of the model, the data is aggregated to reduce the variability of database, which is typical property of lower territorial units. Then the hierarchical structure is built and identification of outlier is also included. The resulting model is calculated with current balance to ensure the validity of the hierarchical structure.

A system of decomposition was designed to solve this mathematical model. The principle lies in the step-by-step solution of individual parts, as shown in Fig. 1. This heuristic approach can be used for hierarchically structured data from different areas. In the case study, it was successfully applied to the municipal waste production data in the Czech Republic.

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