SPIRAL EXTRUSION DIE DESIGN USING MODIFIED DIFFERENTIAL EVOLUTION ALGORITHM

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Abstract
In this work, a spiral extrusion die for industrial production of plastic foil has been designed using a modified differential evolution algorithm. The proposed method managed to provide a die design that was compliant with all demands of the foil manufacturer and lowered the production cost. Third-Party software is used to compute the die characteristics from the geometry designed by modified differential evolution.

Keywords: spiral die, extrusion, industrial application, differential evolution, heuristic computing.

1 Introduction
The extrusion of plastic and polymer materials [1,2,3] has changed our everyday lives in many aspects. It allowed mass production of various plastics products, structure, and constructions. A spiral extrusion die is nowadays the crucial part of many manufacturing processes. However, with the increasing demand on the cost effectiveness of the production, energy savings, and other demands, the process of designing a satisfactory spiral die is becoming very complex and computationally expensive.

In this work, the challenge set out by a manufacturing company was to find a valid design of a spiral die for an industrial application of plastic foil production. The typically used (deterministic) methods were unable to obtain a valid solution. Therefore, an innovative approach, based on metaheuristics, was used.

Metaheuristics, notably the evolutionary computational techniques are a modern tool for fast and effective solving of complex optimization problems with high computational demands and complex fitness search space. Among the many popular metaheuristics, the Differential Evolution (DE) algorithm [4,5,6,14] holds a prominent place as one of the most consistently well-performing methods. Recent examples of successful applications of differential evolution for solving the real-world problem were reported in [7], [8] or [9].

In this work, a modified differential evolution algorithm is proposed and utilized alongside a third-party modeling software to design a spiral die for an industrial application.

The rest of the paper is structured as follows: The next section introduces the spiral extrusion die and the modeling software used in this work. The tunable parameters of the design are described in section three. The cost function is defined in section four. The proposed modified differential evolution algorithm is described in section five. Following are the results discussion and the conclusion.

2 The Spiral Extrusion Die

In this work, a spiral extrusion die for industrial production of plastic foil (cling film) was designed. During the procedure of foil production, the material is extruded thru the channels on the spiral die. The depth of the channels decreases in the direction to the reference end of the die. The spiral die is mainly popular for its low-pressure demands and good melt distribution.

The spiral die allows production from various materials and in many conditions. A depiction of a spiral extrusion die is given in Fig. 1 alongside with cross-section schematics that depicts the flow of the material in the die. The material flows from the bottom part of the die to the top, following the spiral.
2.1 The Model

The Virtual Extrusion Laboratory software module “Spiral Die” [13] was used to produce the extrusion die characteristics. A visualization of the pressure change in the spiral die created by the Virtual Extrusion Laboratory software is depicted in Fig. 2. The software provides the extrusion die characteristics based on a provided geometry of the die. Finding the optimal die geometry was the aim of this work.
3 Spiral Die Geometry Parameters

The spiral die geometry is heavily constrained for both manufacturing and functional reasons. That leaves only four tunable parameters for the optimizer. Despite the low-dimensionality of the problem, the search-space is very complex, and the design optimization is very computationally expensive. The four tunable parameters that were optimized in this work are described further here.

3.1 The Die Input and Output Gap

The output gap $G_{oe}$ (see Fig. 3) is the size of the gap between the body and mandrel of the die at the top end (output) of the die. Similarly, the input gap $G_{io}$ is measured at the opposite, lower opening (input) of the die. The size of the input gap $G_{io}$ is mainly affecting the speed of material dispersion. Both parameters affect the material flow, velocity, and pressure. The defined size range for the input and output gap is 0.5–5 mm.

3.2 Input Channel Depth

This parameter $d_{cc}$ represents the depth of the channel $dc$ (Fig. 4) measured at the input opening (reference start) of the spiral die. The defined range for this parameter is 1 – 56 mm.

![Figure 3: Cross-section view of the flow channel](image)

3.3 Channel Radius

This parameter $R_e$ defines the radius of the spiral channel on its whole length (see Fig. 5). It is a critical parameter for the flow and volume of the melted material. The defined range for the radius value is 1–6 mm.
4 Cost Function Definition

During the complex procedure of designing an extrusion die, several criterions need to be taken into consideration and assigned a priority value (weight). The quality of the die is primarily dependent on the distribution of volume (material) flow $Q_v$ of the melted material on the reference end of the die. Another criterion is a low-pressure loss $P_d$ and an even material distribution, judged by the release speed $V_t$. A penalization variable is the shear stress $S_\tau$. The value of shear stress at several points in the die should not drop under a given threshold value. Following is a more detailed description of this criterions and quality indicators. All described values are obtained from the above mentioned commercial solver, based on the proposed geometry of the spiral die.
4.1 The Pressure Loss

The model computes the pressure loss \( P_a \) in MPa. The value is typically in units or dozens of MPa. The goal for the design is to minimize the pressure loss. Minimizing the pressure helps avoid undesirable heating of the die and other parts of the machinery. Further, machinery that allows higher working pressures is generally more costly and has increased energetic demands.

4.2 Output Volume Flow

The value of the output volume flow is computed as a variance value \( \sigma^2 \) (1) of material flow in several output points at the output end of the spiral die,

\[
\sigma^2(Q_v) = \sum_{n=1}^{N} (Q_{vm} - E(Q_v))^2
\]

where \( Q_{vm} \) is a vector of the measured flow values on the output, and \( E(Q_v) \) is the mean flow value (arithmetical average). The goal for the optimizer is to minimize this value. As mentioned above this parameter has the most significant impact on the quality of the design.

4.3 Channel Release Velocity

The speed of material flow at the release points. This value is given in mm/s. The quality criterion is the sum of squared differences \( E_v \) of the calculated release velocity \( V_{ln} \) and demanded release velocity \( V_{ln_0} \) value at each release point (2).

\[
E_v(V_l) = \sum_{n=1}^{N} (V_{ln} - V_{ln_0})^2
\]

In ideal case, the release velocity change at different points will follow a linear curve.

4.4 Shear Stress

The shear stress value is measured (in Pa) at several measuring points on the spiral die. The value should not drop under 30kPa. This value is important for the optimal movement of the material in the die. With too low values of shear stress, the material might start settling inside the die and burn. For this reason, a penalization \( P_{ss} \) is introduced into the cost function in the following form (3)

\[
P_{ss} = \frac{30000 - S_x}{1000}
\]

4.5 The Cost Function Completion

All above-described criterions are designed in such fashion that the optimal value is obtained by minimization of the criterions. Therefore, it is possible to complete the cost function as a summation of the criterions \( P_d, \sigma^2 \) and \( E_v \) plus the penalization value \( P_{ss} \). However, most of the input parameters are real physical values that are measured in different units, and decimal multipliers of basic units and a balancing mechanism is needed. Weights are introduced (based on the magnitude of the measured quantities) to allow the summation of the criterions, leading to the cost function \( CF(G_{tx}, G_{ts}, d_{cr}, R_c) \) given by (4),

\[
CF(G_{tx}, G_{ts}, d_{cr}, R_c) = w_1 P_d + w_2 \sigma^2(Q_v) + w_3 E_v(V_l) + P_{ss}
\]

where: \( w_1 = 10^{-7}, w_2 = 10^7 \) and \( w_3 = 10^{-2} \).
5 Differential Evolution

The canonical 1995 DE [4] is based on the idea of evolution from a randomly generated set of solutions of the optimization task called population \( P \), which has a preset size of \( NP \). Each individual (solution) in the population consists of a vector \( x \) of length \( D \) (each vector component corresponds to one attribute of the optimized task) and objective function value \( f(x) \), which mirrors the quality of the solution. The number of optimized attributes \( D \) is often referred to as the dimensionality of the problem, and such generated population \( P \), represents the first generation of solutions.

The individuals in the population are combined in an evolutionary manner in order to create improved offspring for the next generation. This process is repeated until the stopping criterion is met (either the maximum number of generations, or the maximum number of objective function evaluations, or the population diversity lower limit, or overall computational time), creating a chain of subsequent generations, where each following generation consists of better solutions than those in previous generations – a phenomenon called elitism. The combination of individuals in the population consists of three main steps: Mutation, crossover, and selection.

In the mutation, attribute vectors of selected individuals are combined in simple vector operations to produce a mutated vector \( v \). This operation uses a control parameter – scaling factor \( F \). In the crossover step, a trial vector \( u \) is created by selection of attributes either from mutated vector \( v \) or the original vector \( x \) based on the crossover probability given by a control parameter – crossover rate \( CR \). Finally, in the selection, the quality \( f(u) \) of a trial vector is evaluated by an objective function and compared to the quality \( f(x) \) of the original vector and the better one is placed into the next generation.

From the basic description of the DE algorithm, it can be seen, that there are three control parameters, which have to be set by the user – population size \( NP \), scaling factor \( F \) and crossover rate \( CR \). It was shown in [10] and [11], that the setting of these parameters is crucial for the performance of DE. Fine-tuning of the control parameter values is a time-consuming task, and therefore, many state-of-the-art DE variants use self-adaptation in order to avoid this cumbersome task [5,6,12]. In this work, a simple adaptation of \( F \) and \( CR \) parameters is implemented in order to avoid the problem of the correct setup of these parameters, that would be computationally very expensive. The algorithm proposed with such change was titled Auto-Adaptive Differential Evolution (AADE).

5.1 Changes in AADE

The AADE algorithm implements a simple adaptive behavior for scaling factor \( F \) and crossover rate values \( CR \) since those influence the optimization process of the DE. For each mutation and crossover step, \( F_i \) and \( CR_i \) values are generated dynamically for each individual from a normal distribution with the mean value of \( F_m \) or \( CR_m \) and standard deviation of 1 (5).

\[
\begin{align*}
CR_i &= N(CR_m, 1) \quad \text{and} \quad F_i = N(F_m, 1),
\end{align*}
\]

where \( F_m \) and \( CR_m \) are mean values of successful scaling factor and crossover rate parameters, respectively. During the selection step, values of \( F \) and \( CR \) that helped to produce a better offspring are stored in a corresponding memory (\( S_F \) and \( S_{CR} \)). After each generation, the mean values of the contents of these memories are computed and stored into \( F_m \) and \( CR_m \) (6). For the first generation, \( F_m \) is set to 0.5 and \( CR_m \) to 0.8. It is also important to note, that \( F_i \) and \( CR_i \) values are bounded between 0 and 1 and if they are generated outside of that range, their value is set to the closest boundary value.

\[
F_m = \text{mean}(S_F), S_F \neq \emptyset \quad \text{and} \quad CR_m = \text{mean}(S_{CR}), S_{CR} \neq \emptyset.
\]

Since the preliminary testing showed a problem with premature stagnation of the population, the stagnation restart of \( F_m \) and \( CR_m \) parameters were implemented. This restart resets \( F_m \) and \( CR_m \) values to the initial 0.5 and 0.8 respectively after 30 generations without an improvement (30 generations of population stagnation). The original mutation and crossover steps are updated only in a slight change of dynamical \( F_i \) and \( CR_i \) values. These two steps are depicted below.
5.2 Mutation

In the mutation step, three mutually different individuals \(x_{r1}, x_{r2}, x_{r3}\) from a population are randomly selected and combined following the mutation strategy. The original mutation strategy of canonical DE is “rand/1” and is depicted in (7).

\[
v_i = x_{r1} + F_i(x_{r2} - x_{r3})
\]  

(7)

Where \(r1 \neq r2 \neq r3 \neq i\), \(F_i\) is the scaling factor value, and \(v_i\) is the resulting mutated vector.

5.3 Crossover

In the crossover step, mutated vector \(v_i\) is combined with the original vector \(x_i\) to produce the trial vector \(u_i\). The binomial crossover (8) is used in canonical DE.

\[
u_{ij} = \begin{cases} 
v_{ij} & \text{if } U[0,1] \leq CR_i \text{ or } j = j_{rand} \\ 
x_{ij} & \text{otherwise} \end{cases}
\]

(8)

Where \(CR_i\) is the used crossover rate value, and \(j_{rand}\) is an index of an attribute that has to be from the mutated vector \(v_i\) (this ensures generation of a vector with at least one new component).

6 The Spiral Die Design Results

This section presents the results of AADE on an industrial spiral die design. The algorithm setup is given in Table 1.

<table>
<thead>
<tr>
<th>Algorithm pseudo-code 1: AADE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Set (NP, CR_m = 0.8, F_m = 0.5), and stopping criterion;</td>
</tr>
<tr>
<td>2. (G = 0, x_{best} = {}, \text{ stag_counter} = 0; )</td>
</tr>
<tr>
<td>3. Randomly initialize population (P = (x_{1,G}, \ldots, x_{NP,G});)</td>
</tr>
<tr>
<td>4. (P_{new} = {}, x_{best} = \text{best from population } P;)</td>
</tr>
<tr>
<td>5. while stopping criterion not met</td>
</tr>
<tr>
<td>6. for (i = 1) to (NP) do</td>
</tr>
<tr>
<td>7. stag_counter++;</td>
</tr>
<tr>
<td>8. (x_i,G = P[i];)</td>
</tr>
<tr>
<td>9. Generate (F_i) and (CR_i) by (1);</td>
</tr>
<tr>
<td>10. (v_{i,G}) by mutation (3);</td>
</tr>
<tr>
<td>11. (u_{i,G}) by crossover (4);</td>
</tr>
<tr>
<td>12. if (f(u_{i,G}) \leq f(x_{i,G}) ) then</td>
</tr>
<tr>
<td>13. (x_{i,G+1} = u_{i,G};)</td>
</tr>
<tr>
<td>14. (F_i \to SF, CR_i \to SCR;)</td>
</tr>
<tr>
<td>15. stag_counter = 0;</td>
</tr>
<tr>
<td>16. else</td>
</tr>
<tr>
<td>17. (x_{i,G+1} = x_{i,G};)</td>
</tr>
<tr>
<td>18. end if</td>
</tr>
<tr>
<td>19. (x_{i,G+1} = P_{new};)</td>
</tr>
<tr>
<td>20. end for</td>
</tr>
<tr>
<td>21. Compute new (F_m) and (CR_m) by (2);</td>
</tr>
<tr>
<td>22. if stag_counter = 30 then</td>
</tr>
<tr>
<td>23. Reinitialize (F_m) and (CR_m;)</td>
</tr>
<tr>
<td>24. end if</td>
</tr>
<tr>
<td>25. (P = P_{new}, P_{new} = {}, x_{best} = \text{best from population } P;)</td>
</tr>
<tr>
<td>26. end while</td>
</tr>
<tr>
<td>27. return (x_{best}) as the best found solution</td>
</tr>
</tbody>
</table>
The proposed method was tested on 30 independent runs. The results overview is presented in Table 2. Further, the convergence of the best run is depicted in Fig. 6. Finally, the best-obtained solution was tested for validity using above-presented criteria.

Table 2: AADE algorithm results (30 runs)

<table>
<thead>
<tr>
<th></th>
<th>Best</th>
<th>Worst</th>
<th>Median</th>
<th>Mean</th>
<th>Std. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>94.6769</td>
<td>175.506</td>
<td>94.6775</td>
<td>110.843</td>
<td>36.1476</td>
</tr>
</tbody>
</table>

![Figure 6: The solution convergence of the proposed method (best run)](image1)

Graphical visualization of the resulted characteristic is presented alongside a commentary. Based on the provided evidence (Fig. 7 – 9), it is possible to acknowledge the quality and validity of the design.

The shear pressure course inside the spiral channel is given in Fig. 7. It is critical that the pressure does not drop back below the 30 kPa threshold. Therefore, the design is valid.

![Figure 7: The course of shear pressure inside the spiral channel (x – time (s))] (image2)

Channel release velocity is supposed to follow a linear curve in the ideal (unrealistic) case. In Fig. 8, the channel release velocity of the designed die is presented. The course is linear enough for valid die design.

![Figure 8: The course of shear pressure inside the spiral channel (x – time (s))] (image3)
The most significant quality indicator is the uniformity of material volume flow on the output of the die (Fig 9). The achieved variance value $\sigma^2 = 0.675\%$ is excellent and more than satisfactory for this particular application.

![Figure 9: The course of material volume flow with a computed variance of the mean (x – time (s))](image)

Finally, the pressure loss in the designed spiral die was approx. 5 MPa. All parameters are therefore satisfactory, and the design is valid.

7 Conclusion

In this work, an extrusion spiral die for the production of plastic foil was designed using a modified differential evolution algorithm called AADE. The initial experiments with standard deterministic methods (carried out by the manufacturing company) were unsuccessful in producing a valid design. Therefore, a metaheuristic, namely the differential evolution algorithm, was chosen to solve the optimization problem.

However, tuning of control parameters of the standard differential evolution is very computationally expensive and unsuitable for this application, given the computational limitations and time requirements; therefore, a simple adaptive differential evolution was proposed. The proposed method managed to produce high quality and valid design of a spiral die for this industrial application that has been successfully used. The newly produced design works with low pressures and improves the economic aspects of the foil manufacturing process.

The method presented in this paper seems to be very effective for this type of complex, soft constrained (penalized) and computationally expensive real-world optimization problem and supports the usefulness of evolutionary based metaheuristic methods for industrial applications. In future research, the possibilities of using different metaheuristics for this particular problem will be investigated, and a performance comparison will be compiled.

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